

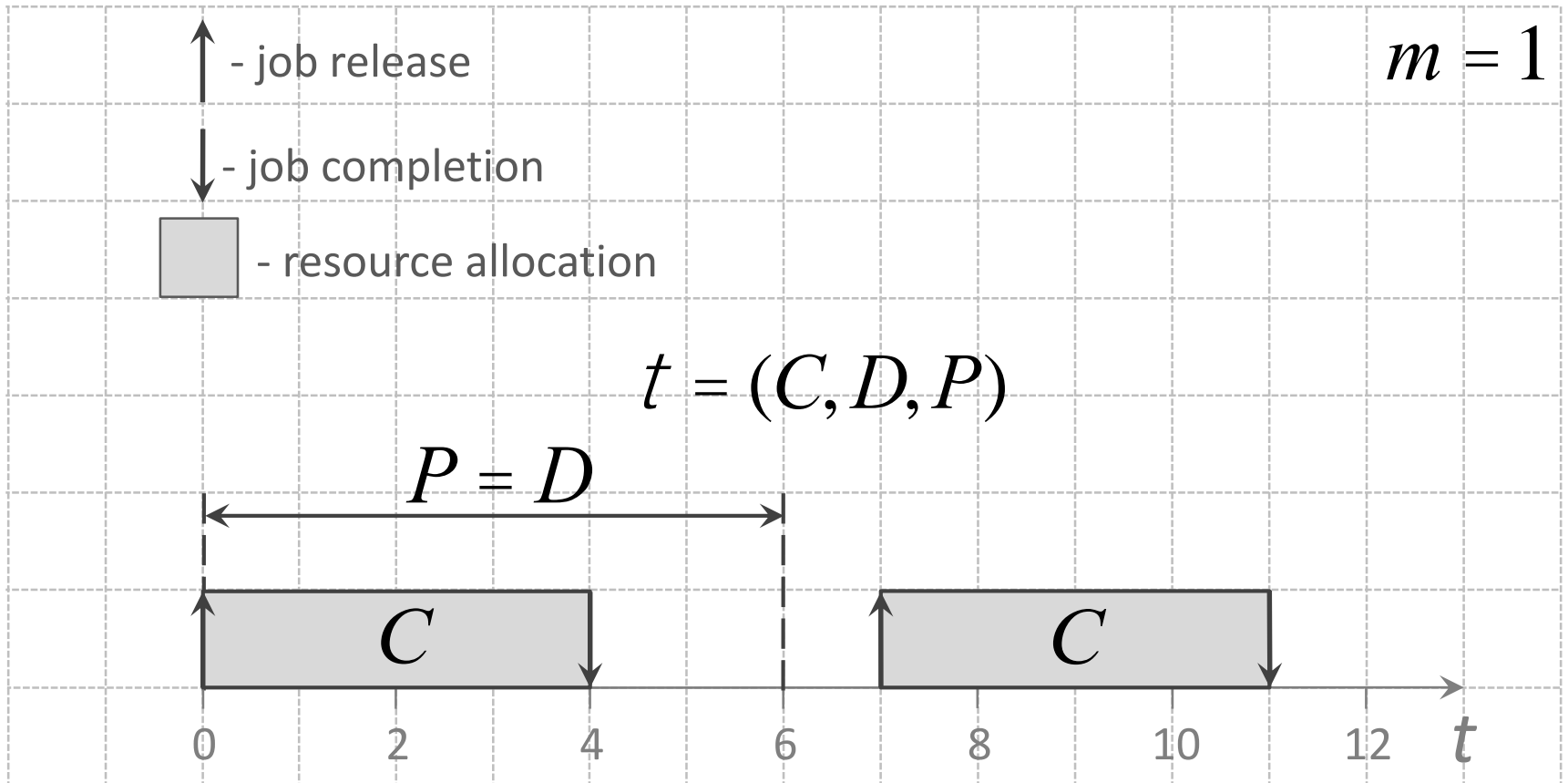
An Exact Schedulability Test for Global FP Using State Space Pruning

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Notation



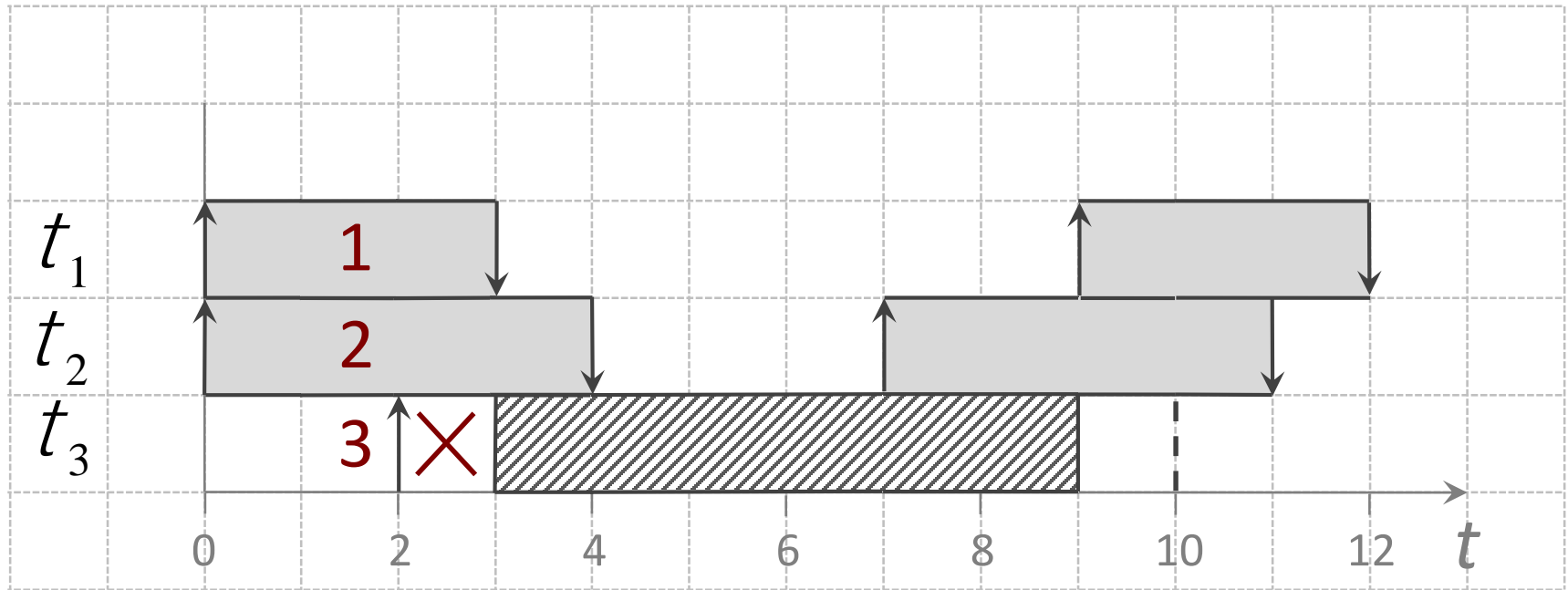
GFP Schedule

$$t_1 = (3, 5)$$

$$t_2 = (4, 6)$$

$$t_3 = (6, 8)$$

$$m = 2$$



- resource available to t_3



- deadline of t_3

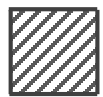
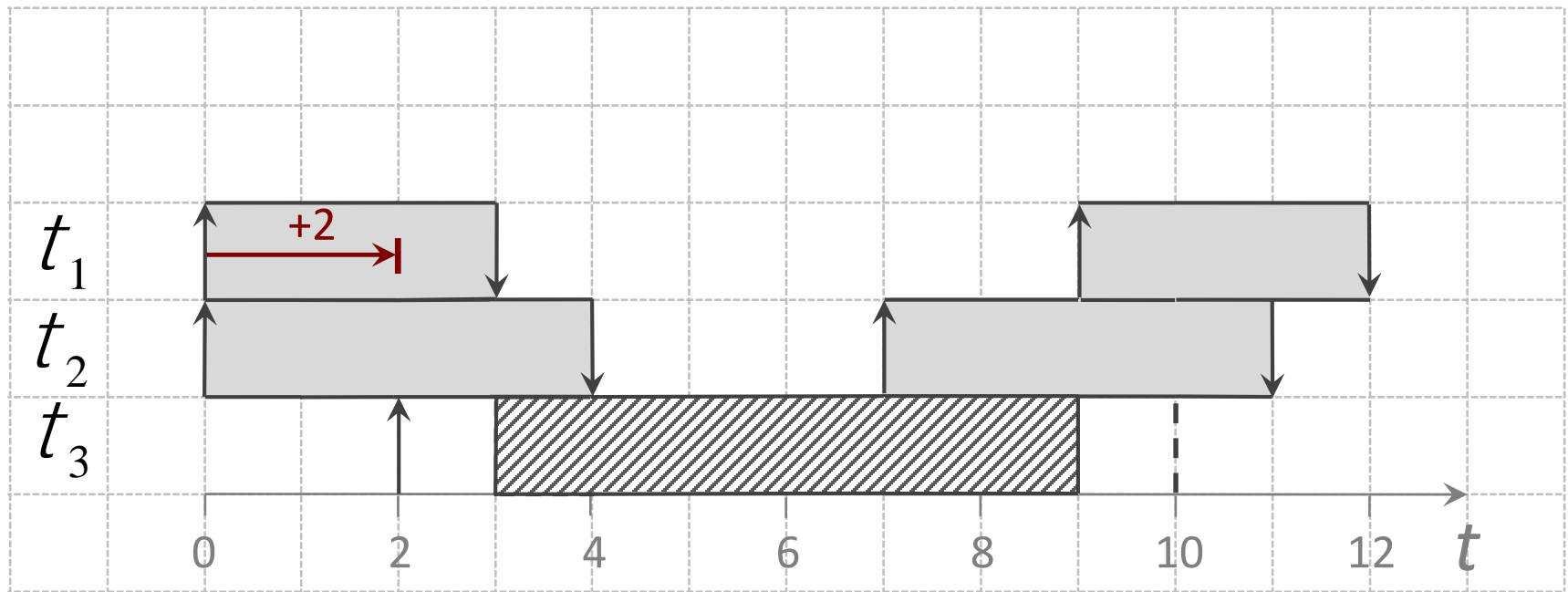
GFP Schedule

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- resource available to t_3



- deadline of t_3

Schedulability Analysis:

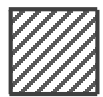
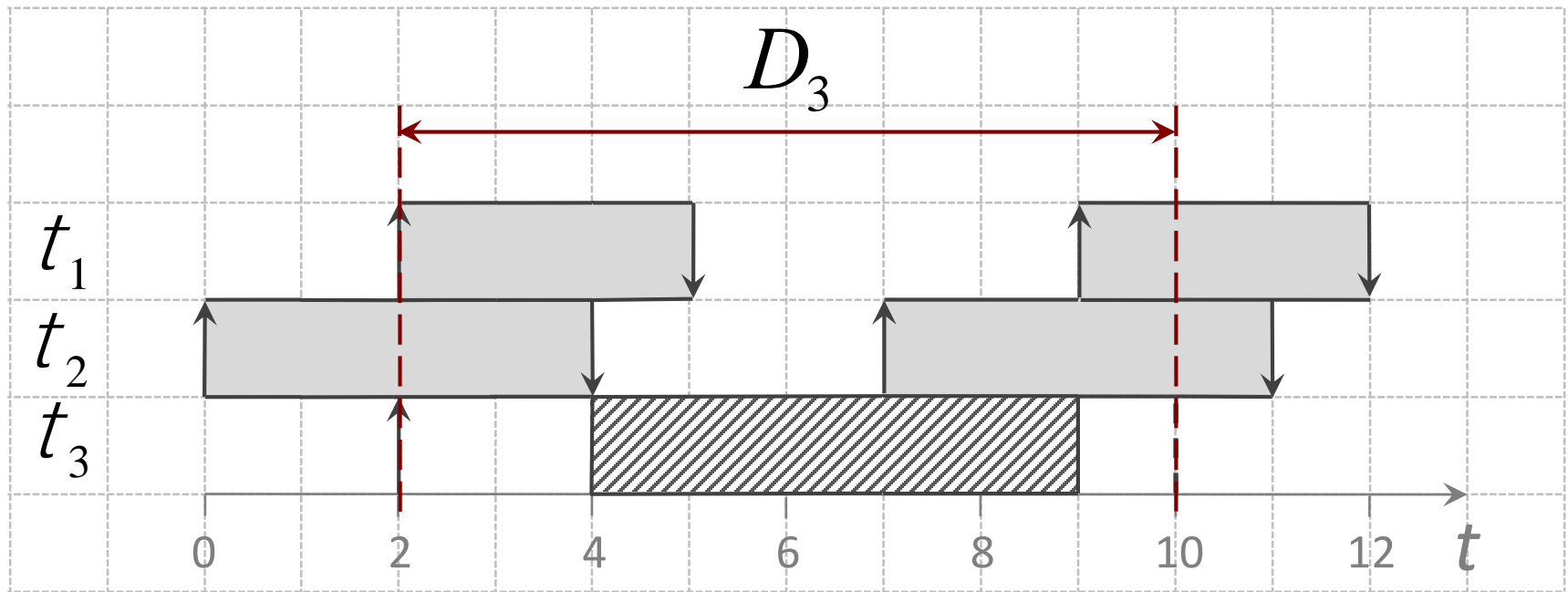
What is the worst-case resource amount?

$$t_1 = (3, 5)$$

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$$t_3 = (6, 8)$$

$$m = 2$$



- resource available to t_3



- deadline of t_3

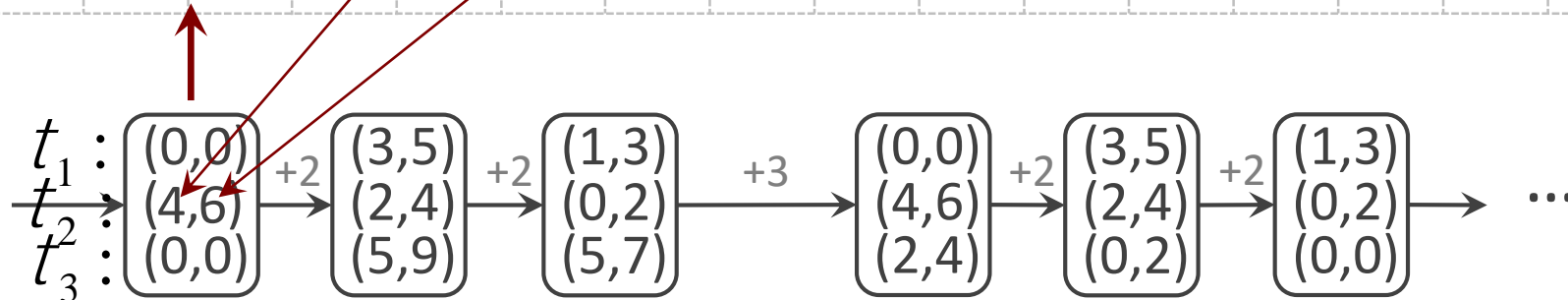
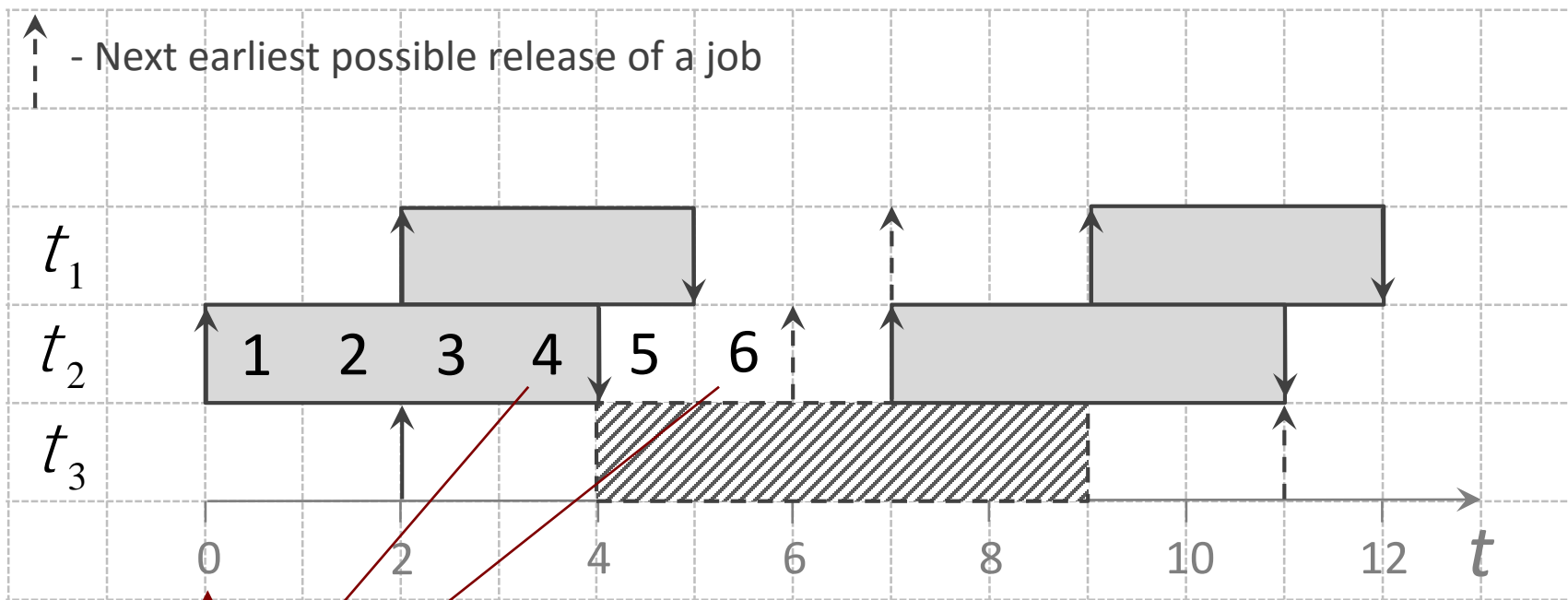
Model of State

$$t_1 = (3, 5)$$

$$t_2 = (4, 6)$$

$$t_3 = (6, 8)$$

$$m = 2$$



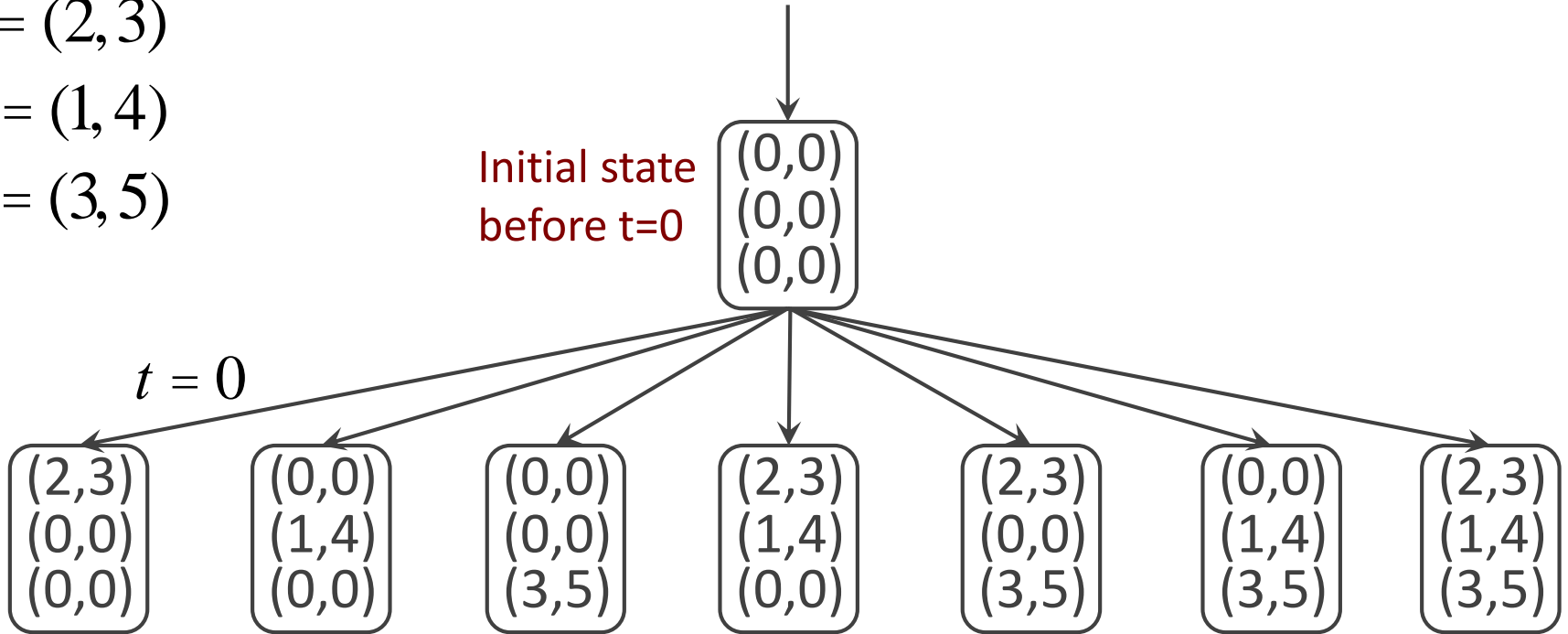
State Transition Graph

[Bonifaci and Marchetti-Spaccamela 2012]

$$t_1 = (2, 3)$$

$$t_2 = (1, 4)$$

$$t_3 = (3, 5)$$



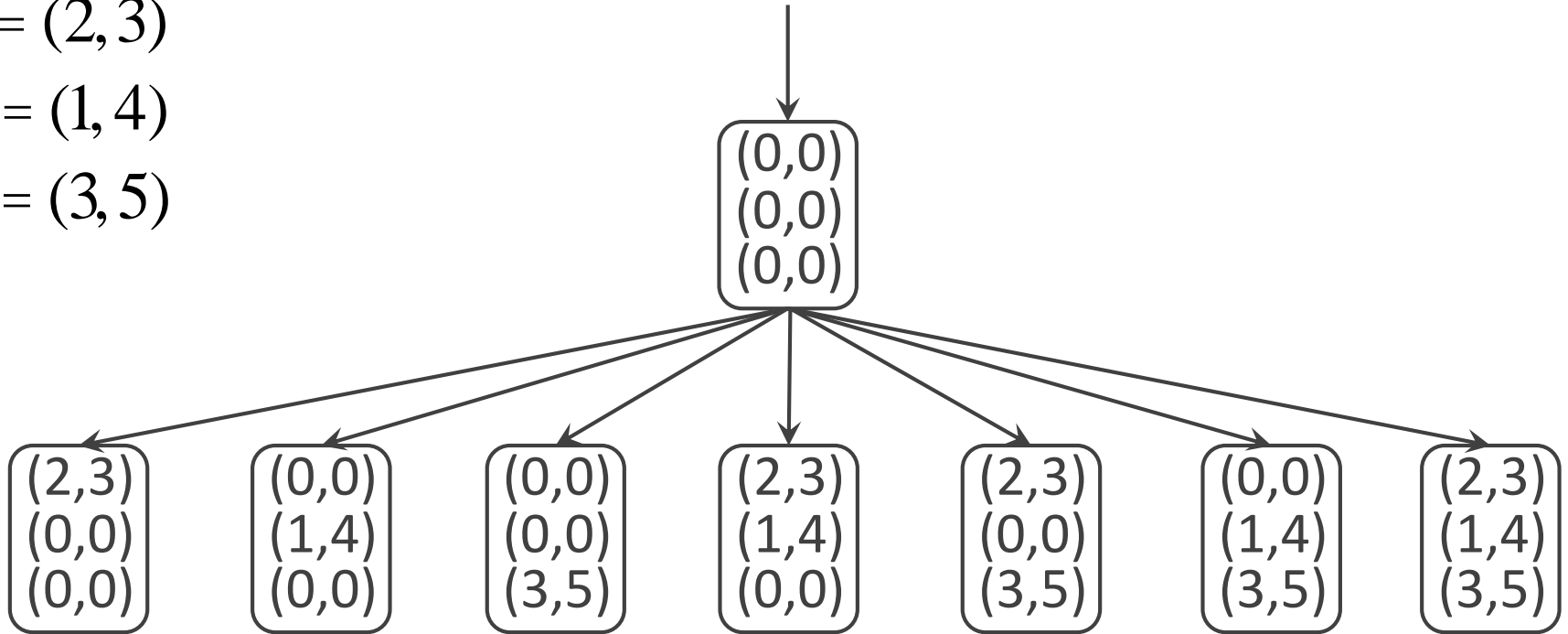
State Transition Graph

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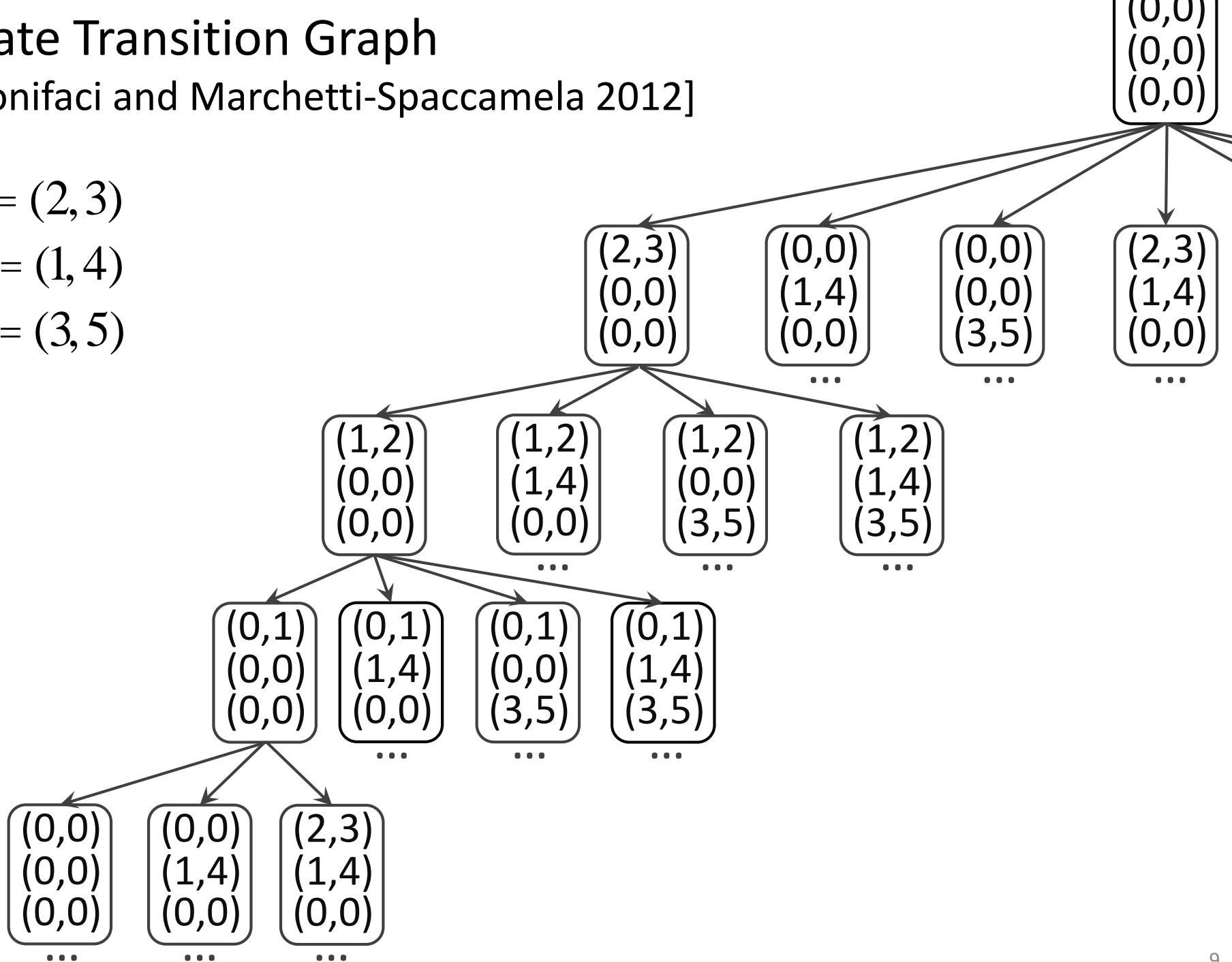
State Transition Graph

[Bonifaci and Marchetti-Spaccamela 2012]

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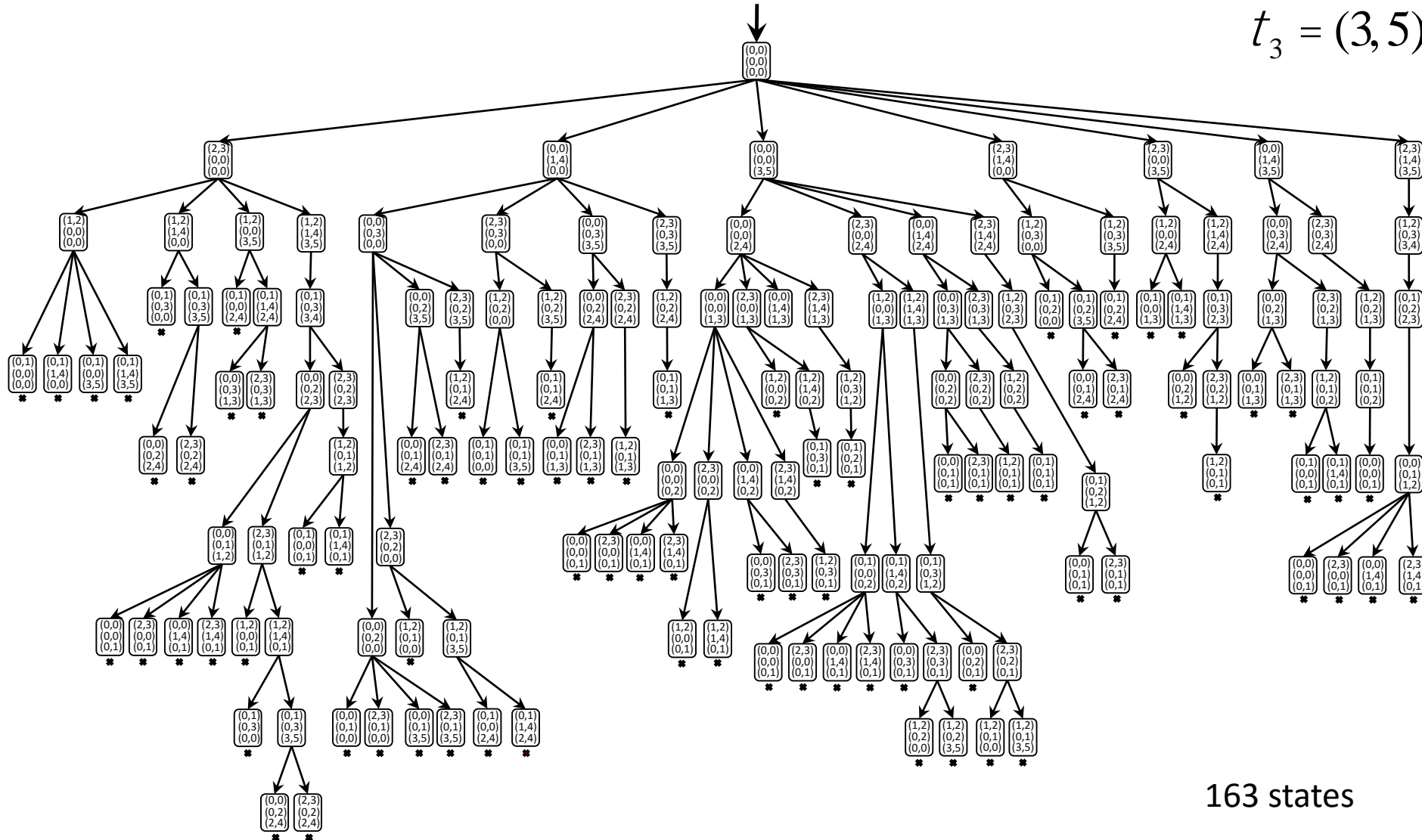
State Transition Graph

[Bonifaci and Marchetti-Spaccamela 2012]

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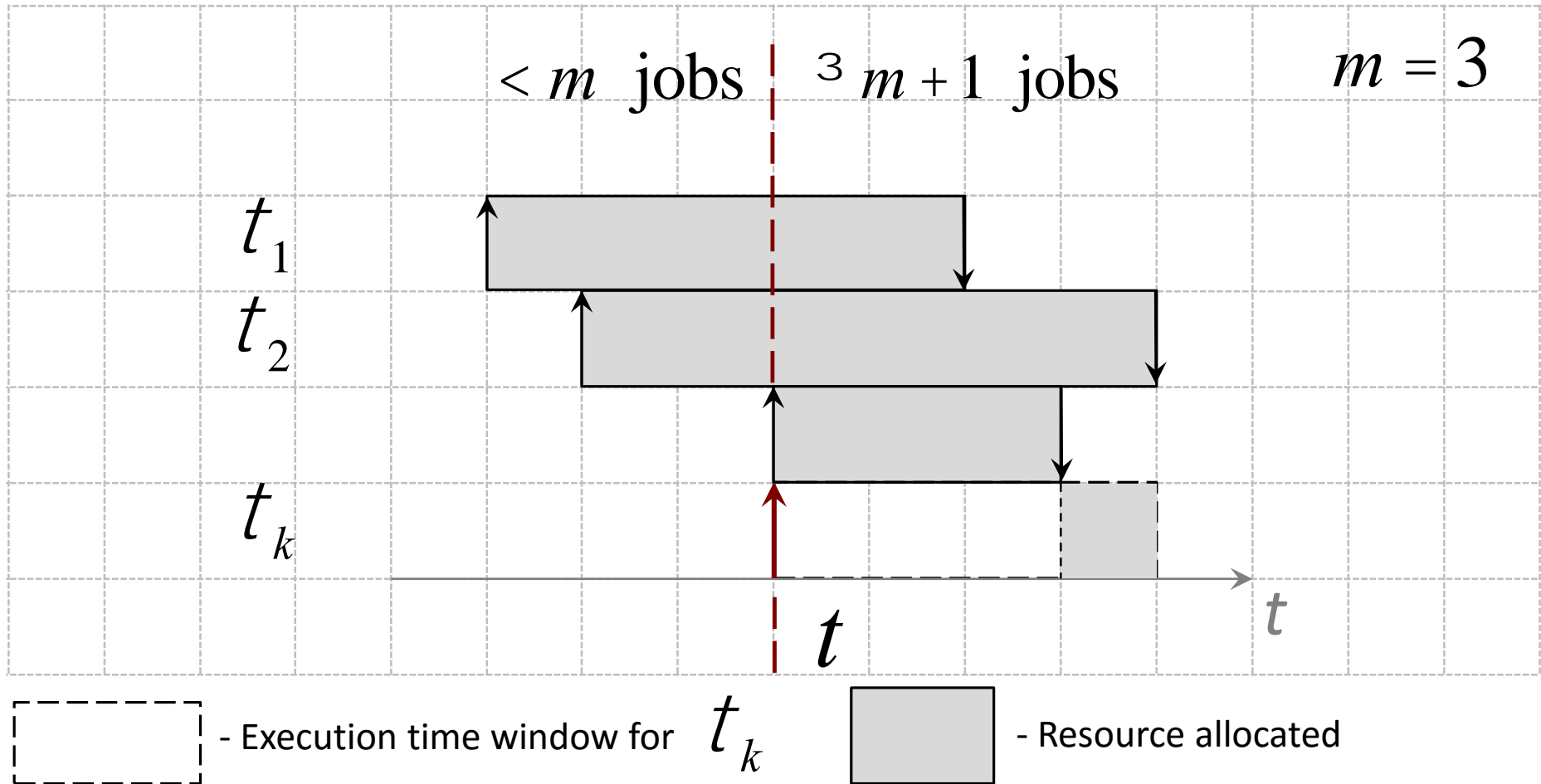
163 states

Schedulability test: Check each state for a deadline miss

Graph pruning: Exclude unneeded states from the analysis

Pruning Constraint 1:

Critical Release Instant [Davis and Burns 2011]



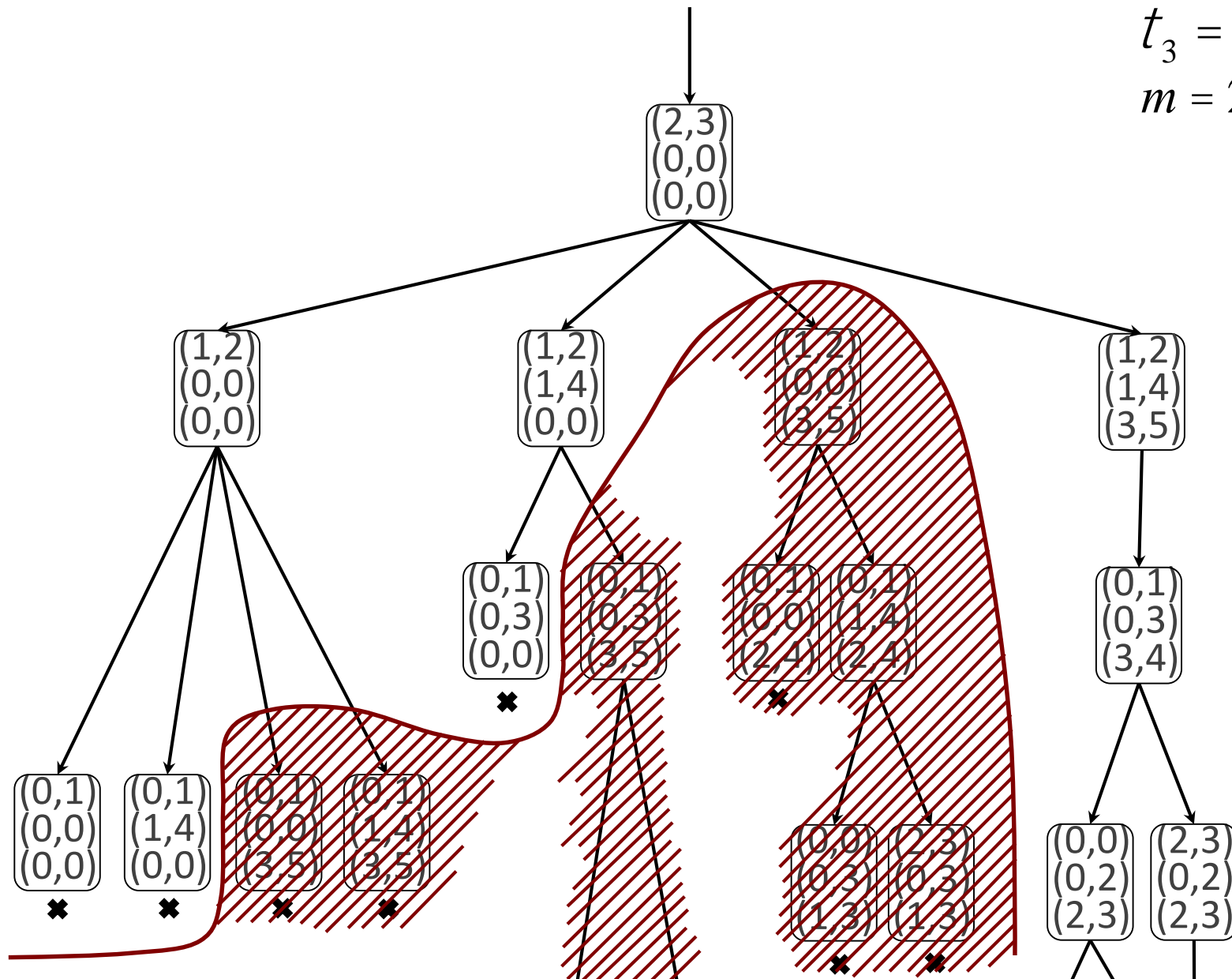
Pruning Constraint 1: Critical Release Instant

$$t_1 = (2, 3)$$

$$t_2 = (1, 4)$$

$$t_3 = (3, 5)$$

$$m = 2$$



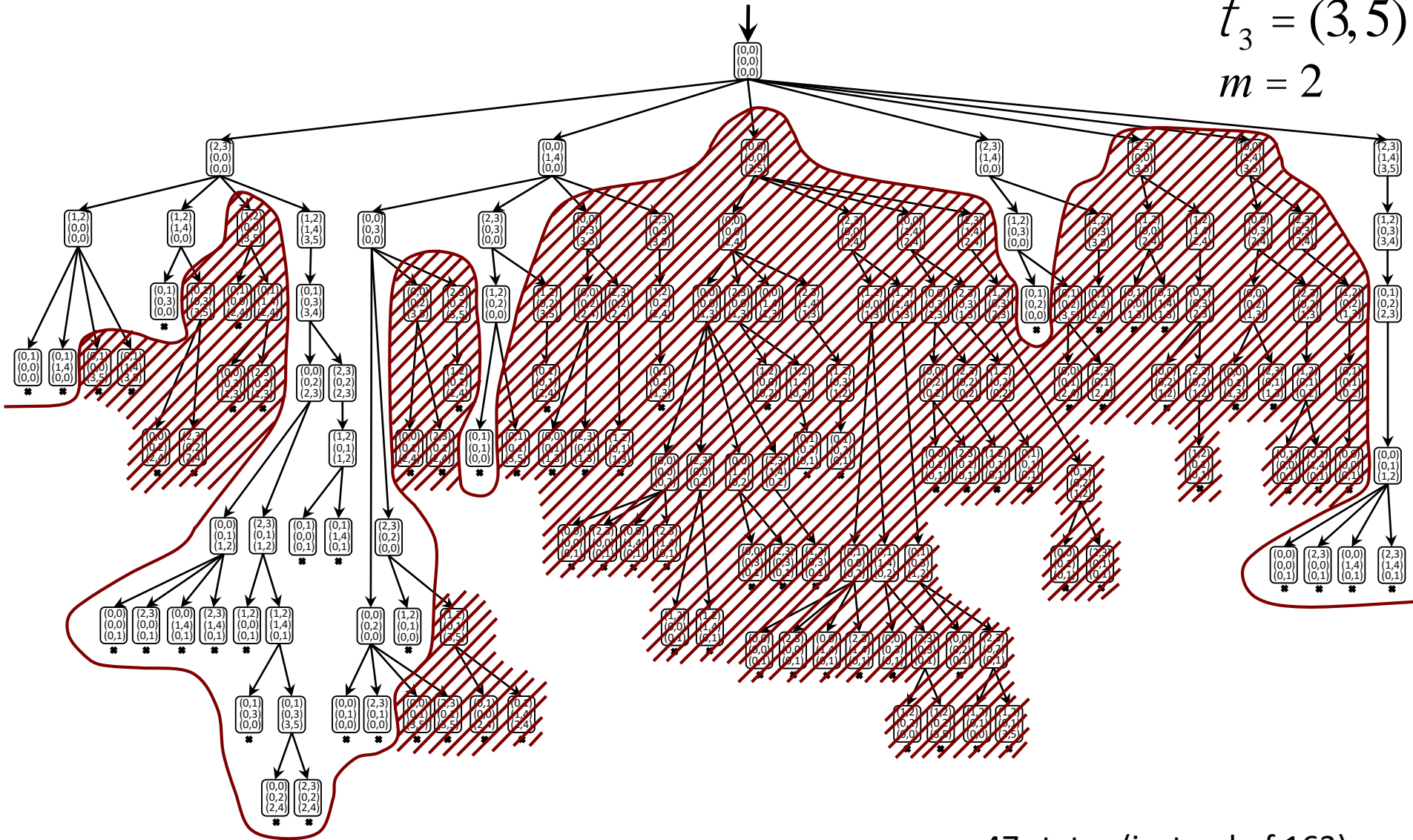
Pruning Constraint 1: Critical Release Instant

$$t_1 = (2, 3)$$

$$t_2 = (1, 4)$$

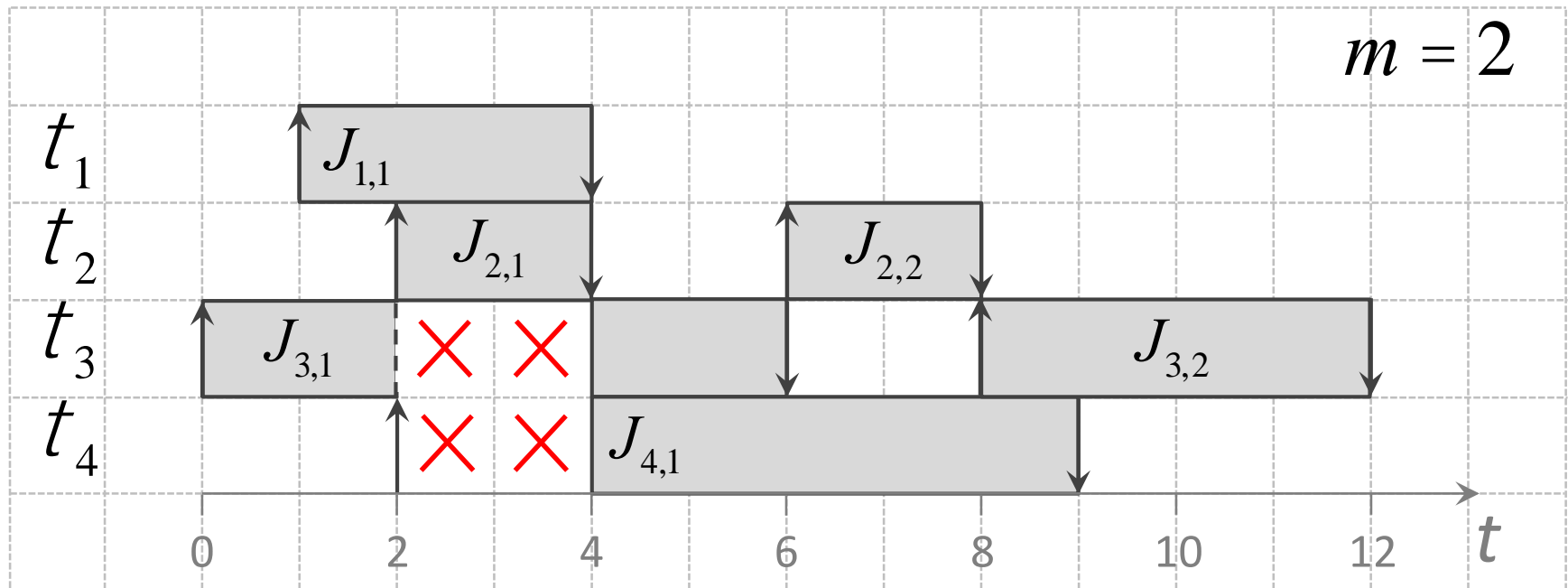
$$t_3 = (3, 5)$$

$$m = 2$$



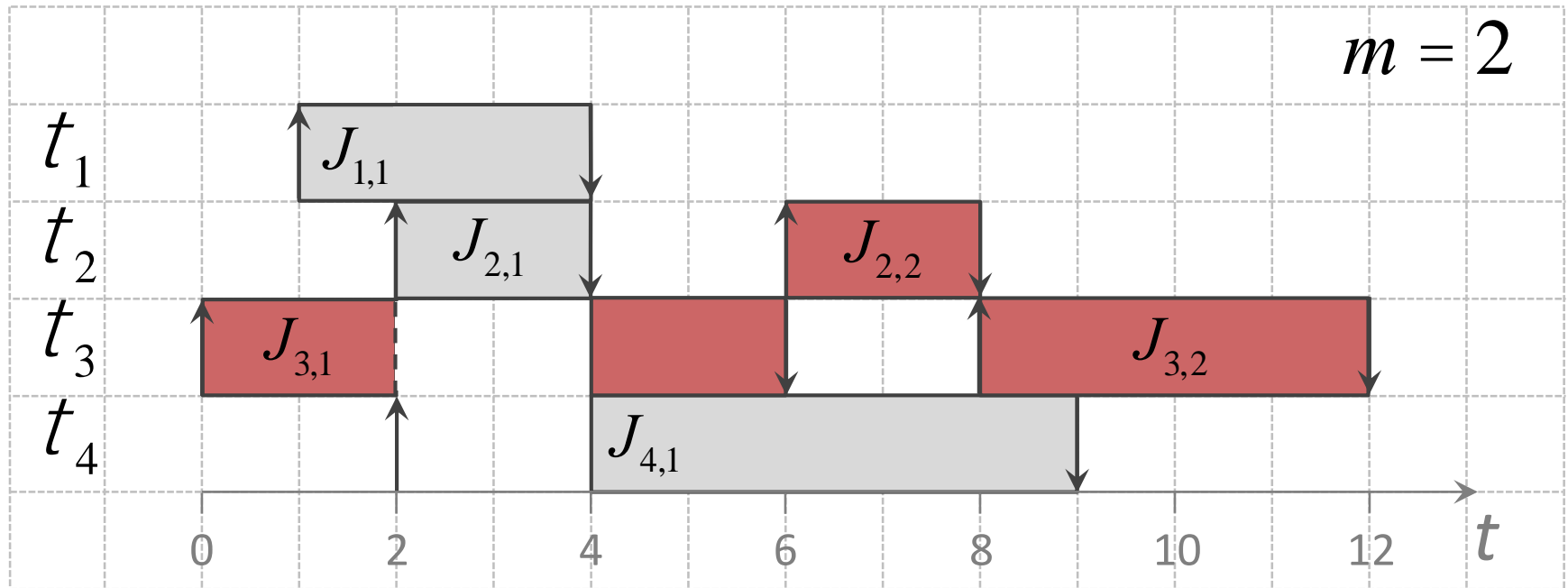
47 states (instead of 163)

Pruning Constraint 2: Job Interference



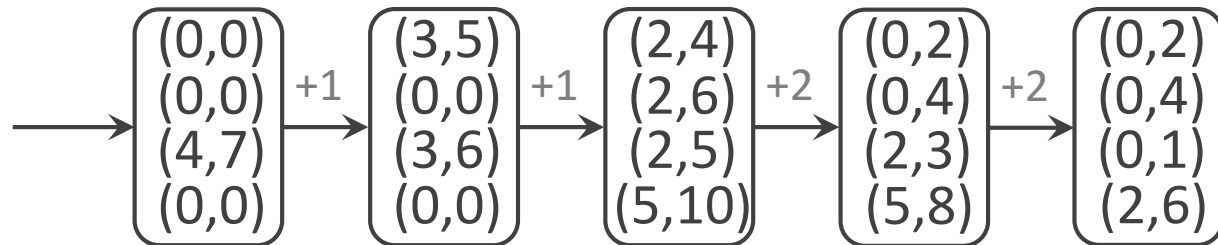
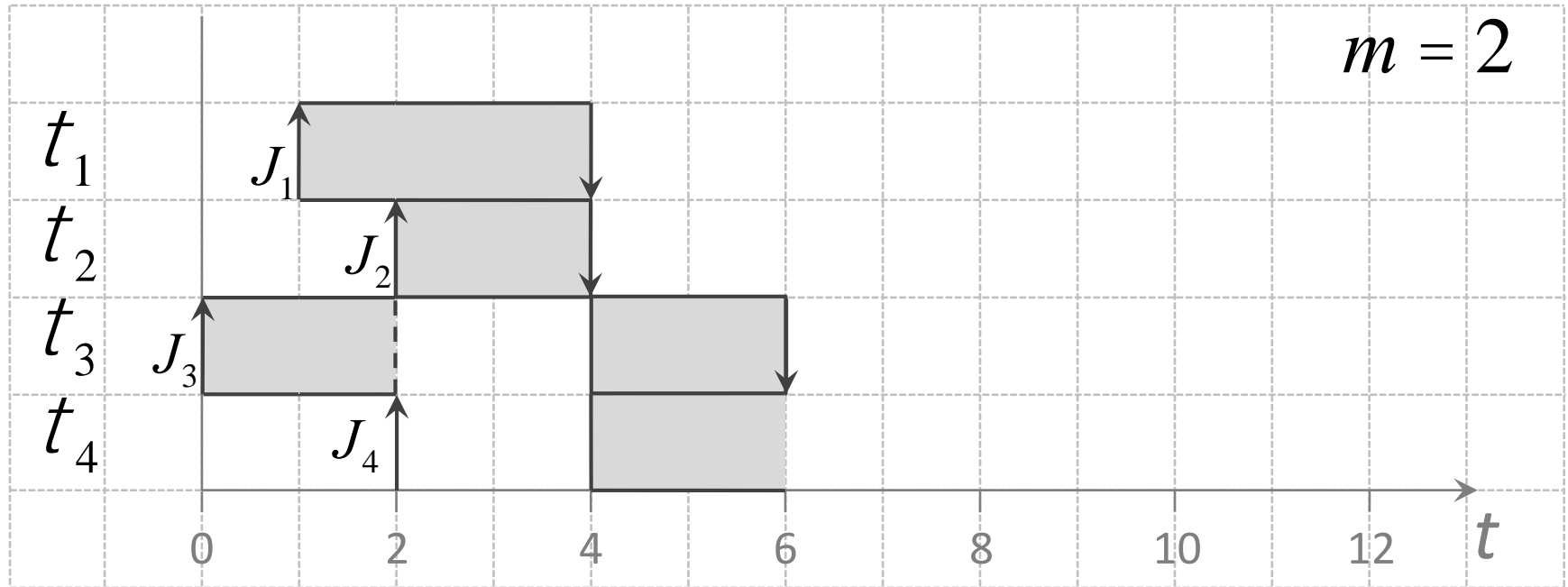
✗ - Job is pending, but no resource allocated

Pruning Constraint 2: Job Interference

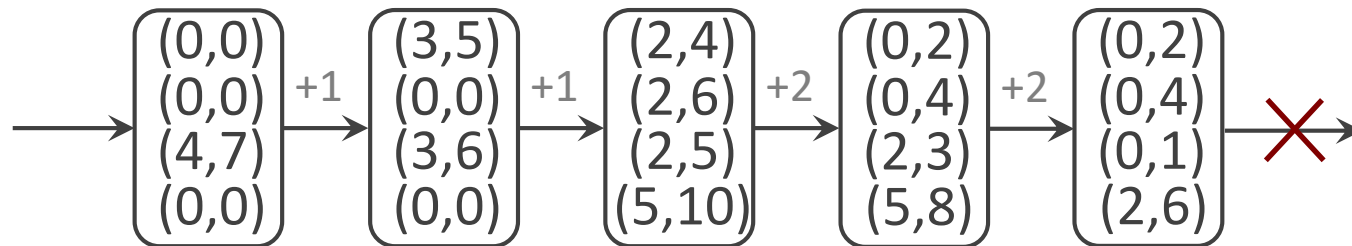
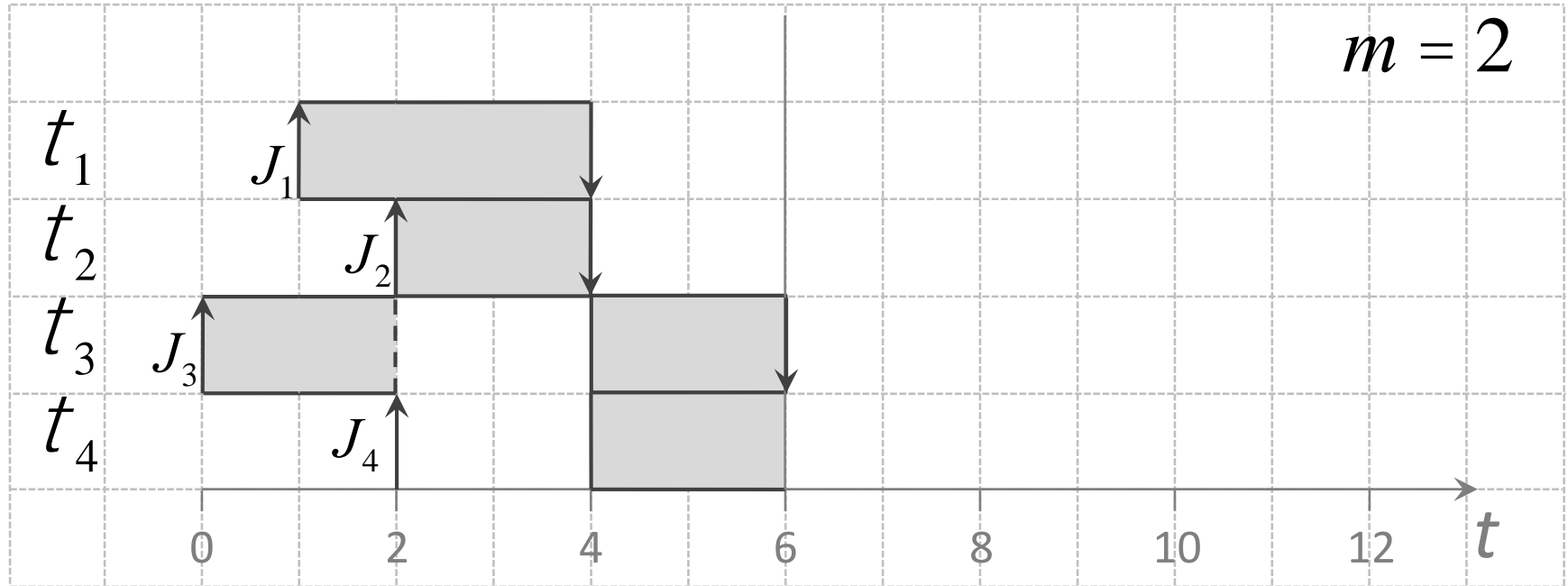


All jobs, not interfering with lower-priority ones, can be removed

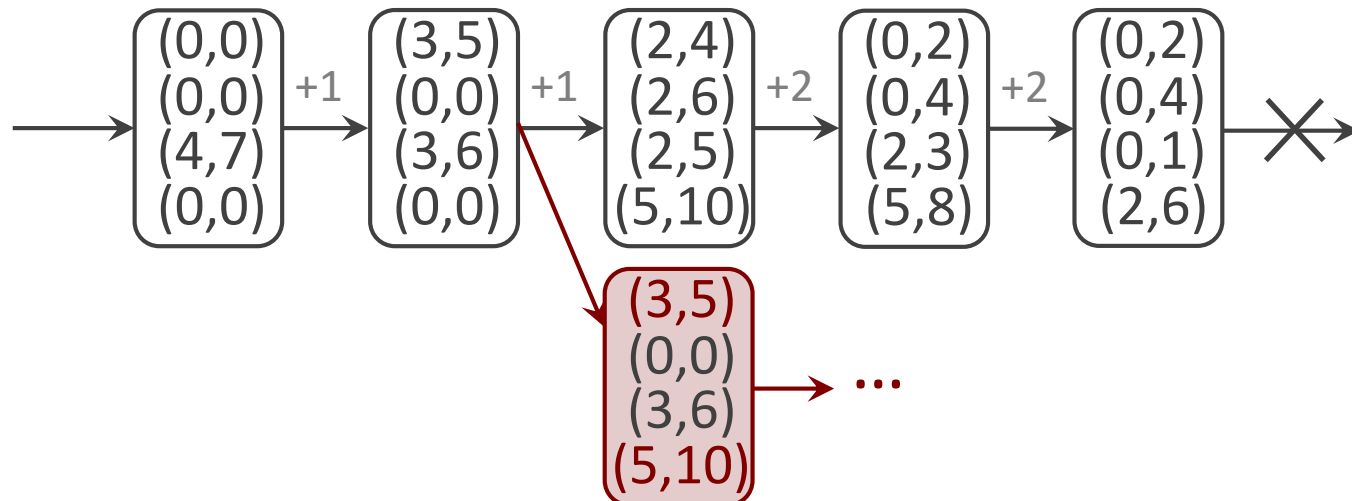
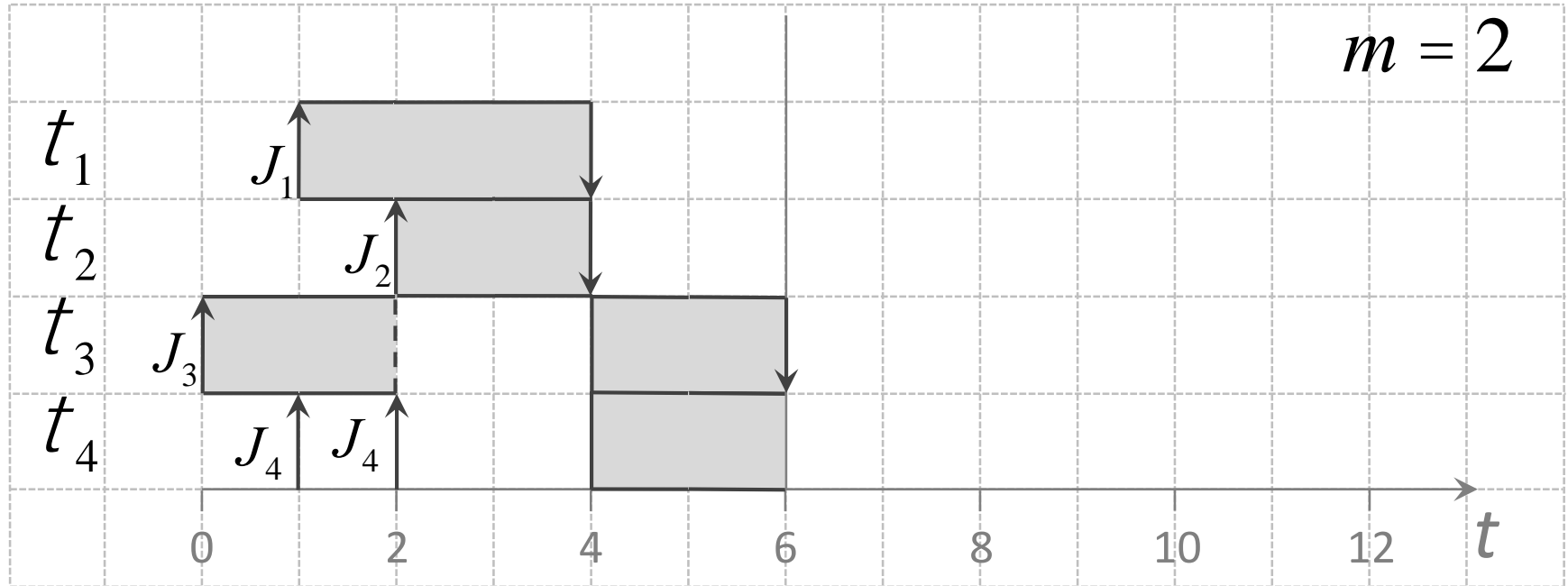
Pruning Constraint 2: Job Interference



Pruning Constraint 2: Job Interference

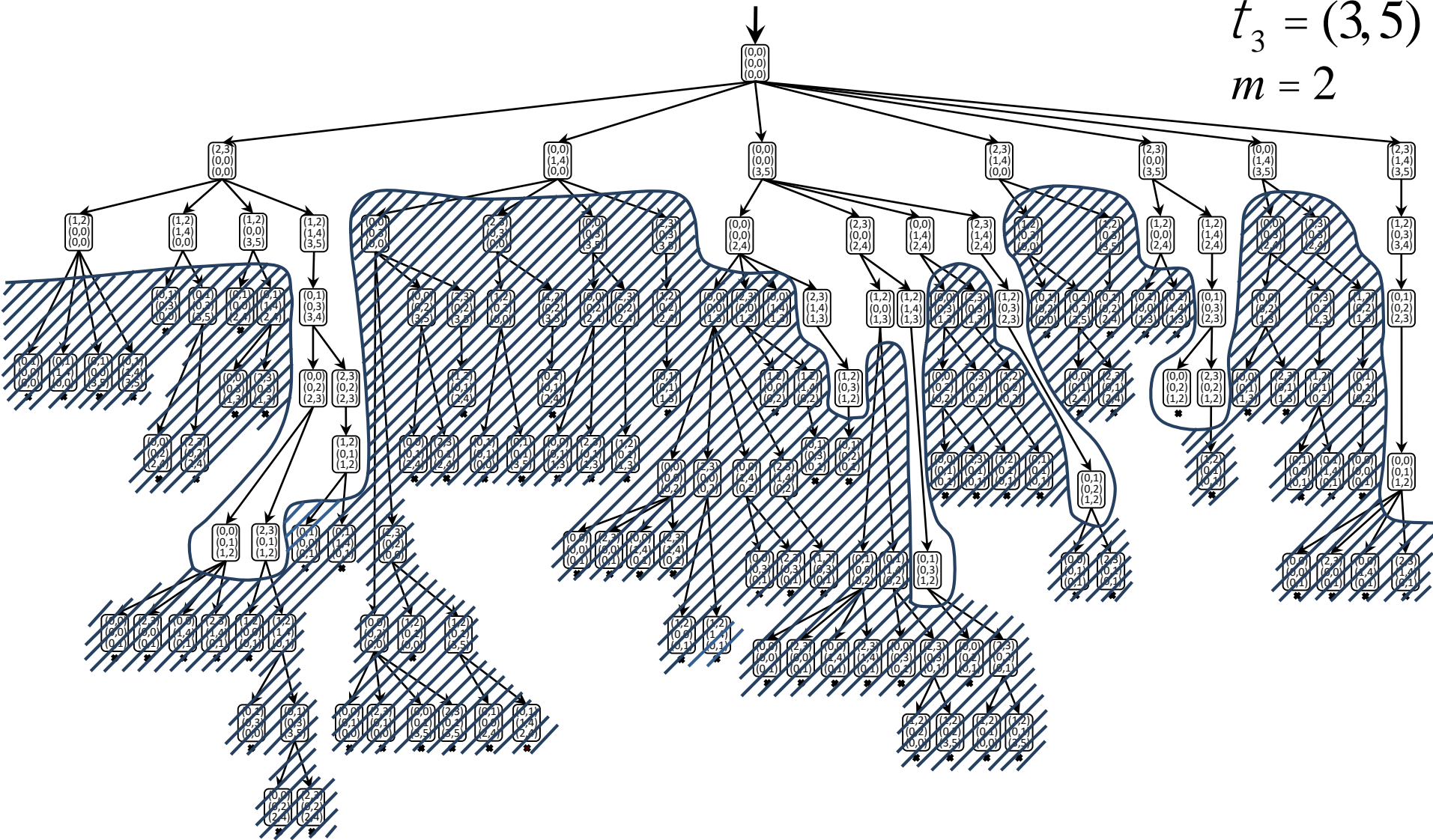


Pruning Constraint 2: Job Interference



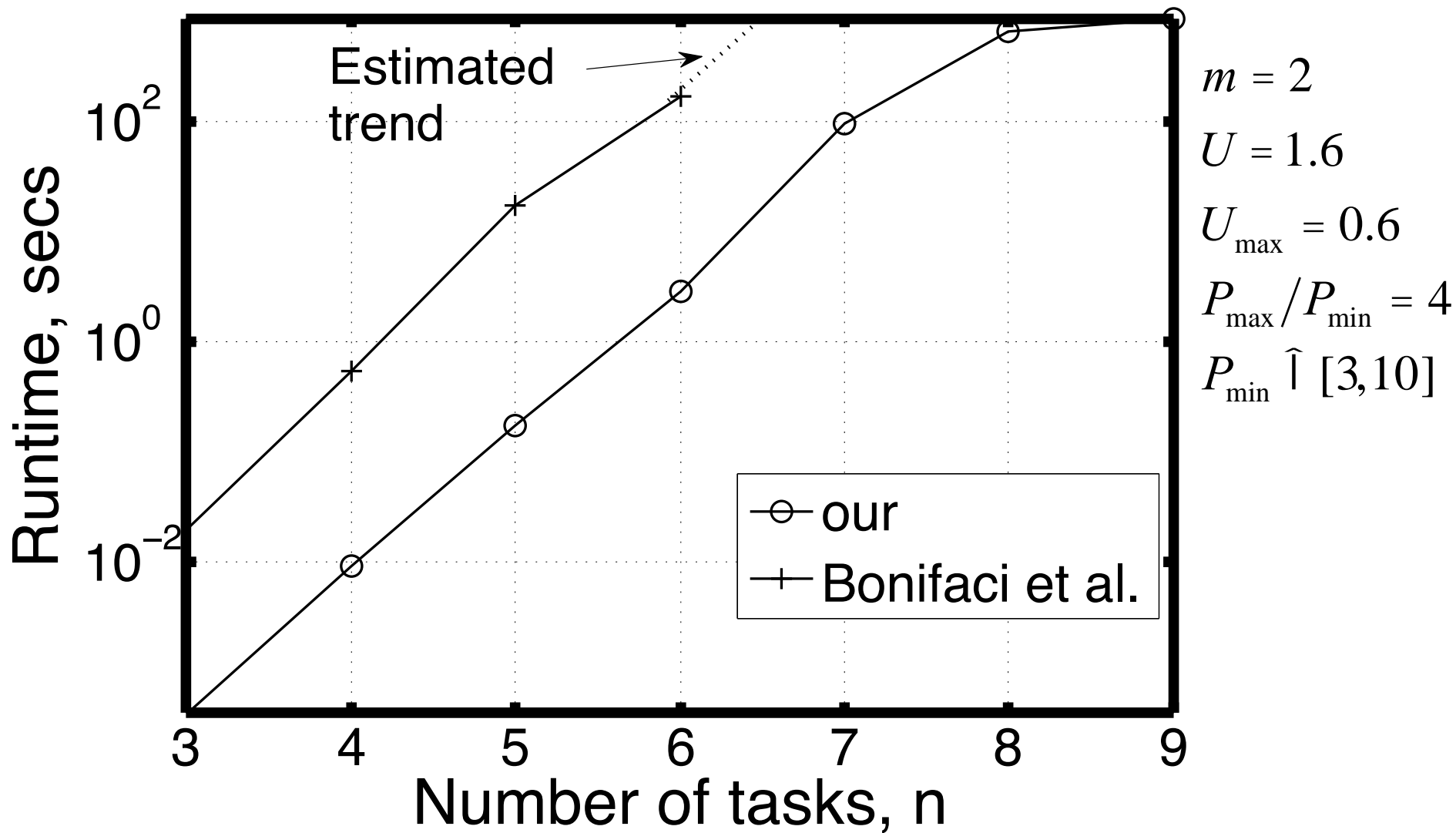
Pruning Constraint 2: Job Interference

$t_1 = (2, 3)$
 $t_2 = (1, 4)$
 $t_3 = (3, 5)$
 $m = 2$

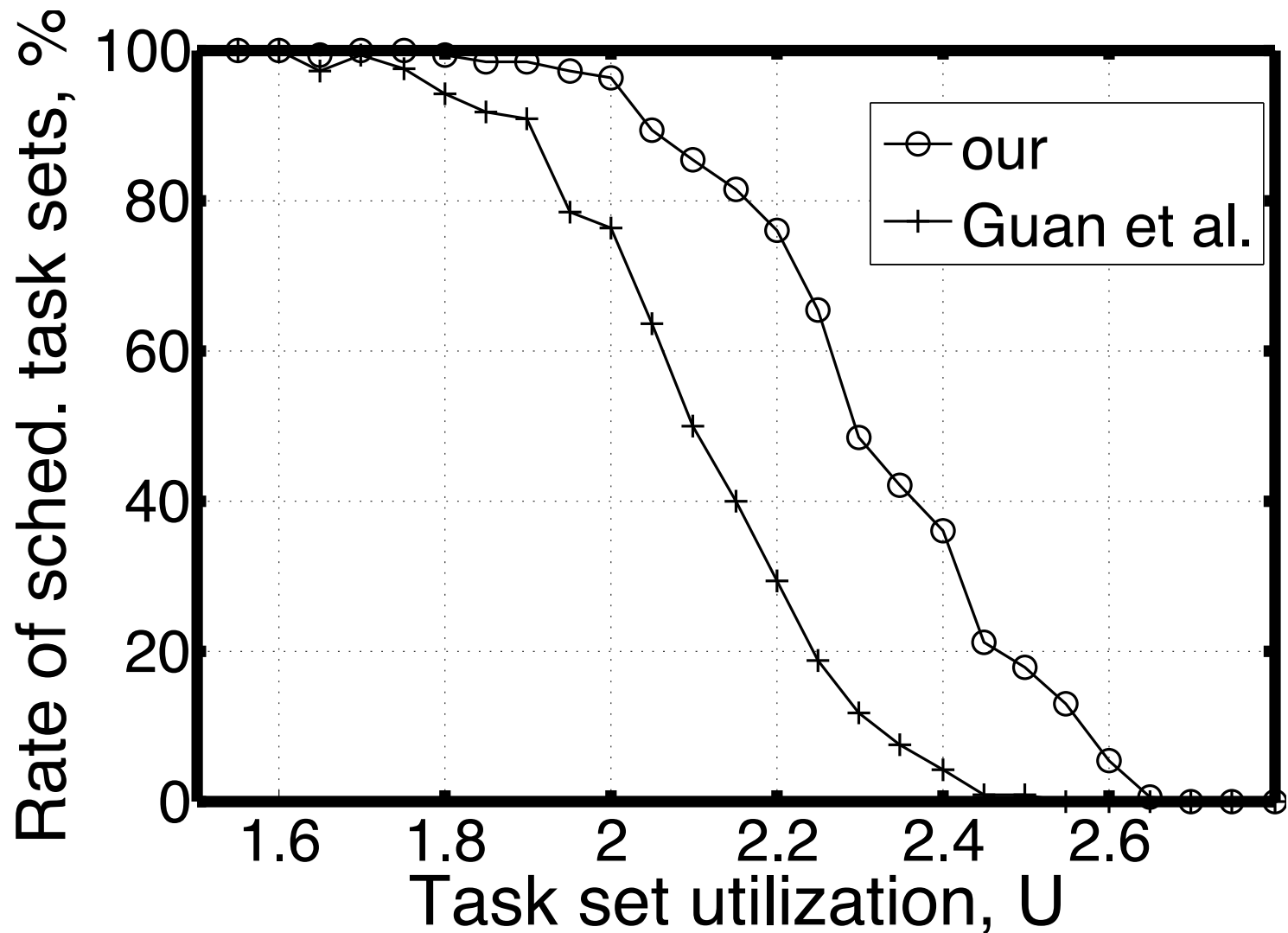


35 states (instead of 163)

Runtime Evaluation



Exact vs. Sufficient Tests



$m = 2$

$n = 7$

$U_{\max} = 0.6$

$P_{\max} / P_{\min} = 4$

$P_{\min} \hat{=} [3, 10]$

Conclusion

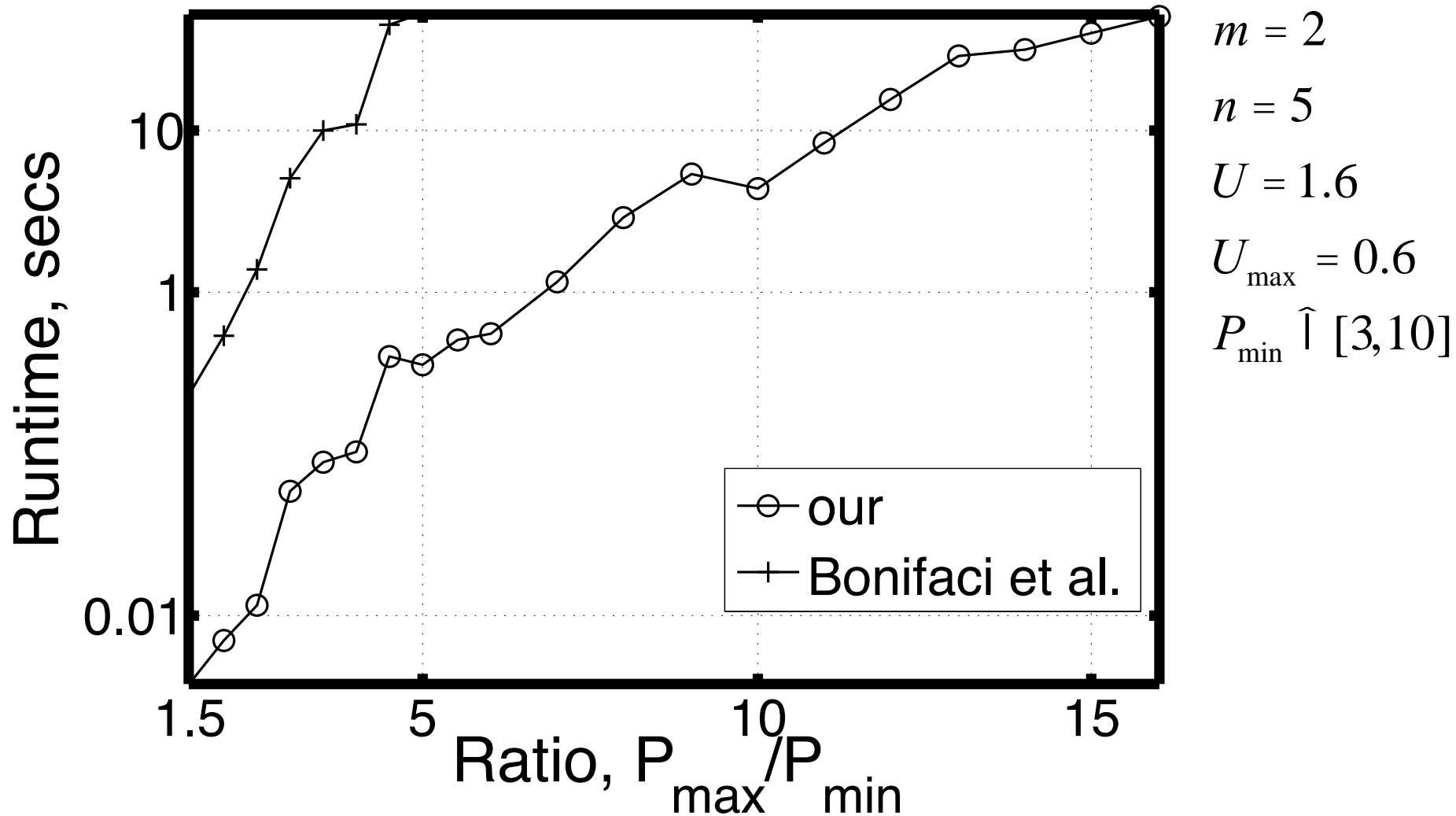
Pruning reduces the runtime of the exact schedulability test

Runtime increase remains exponential for the number of tasks

Exact test remains not scalable for realistic task sets

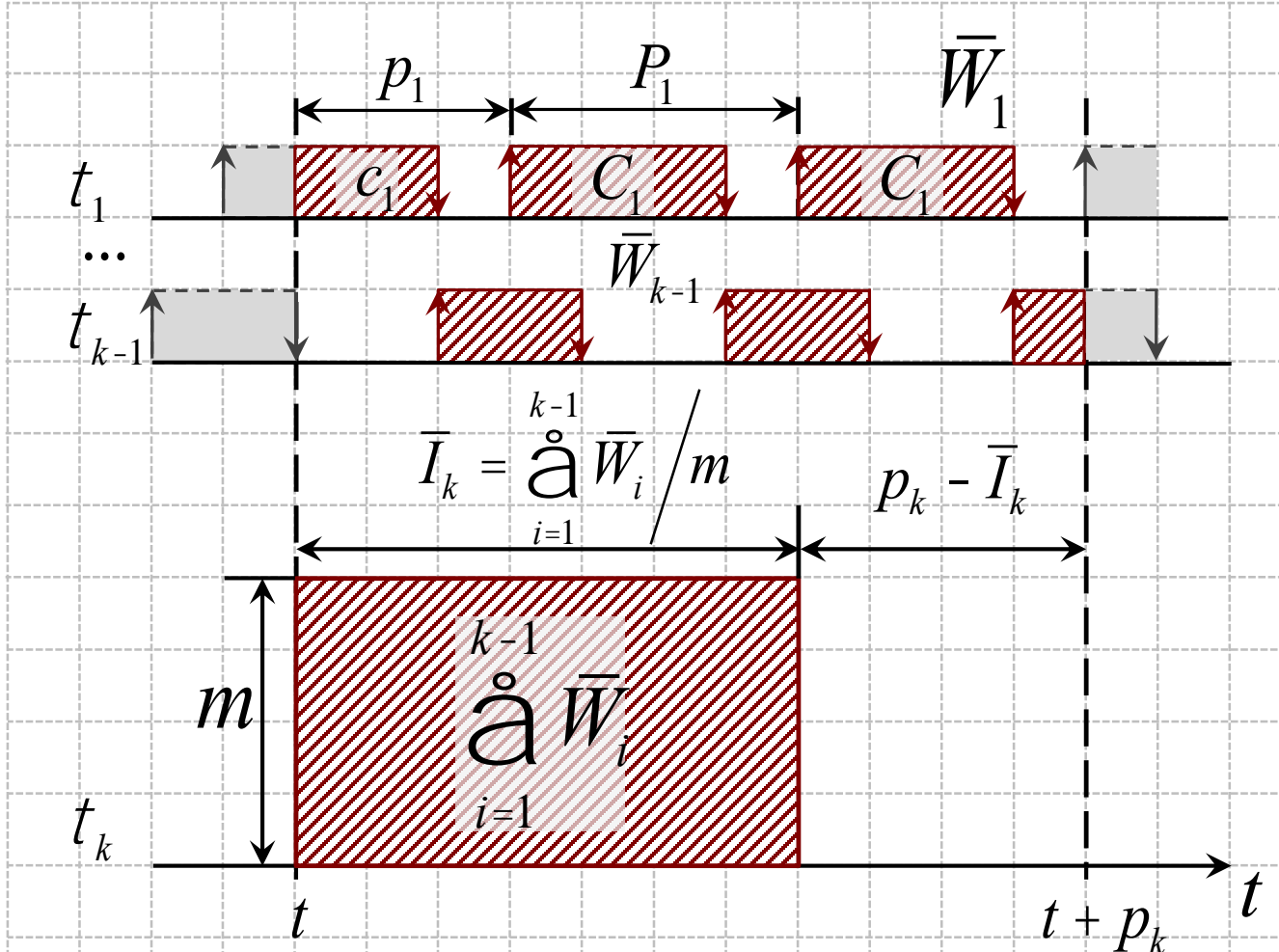
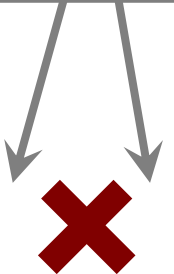
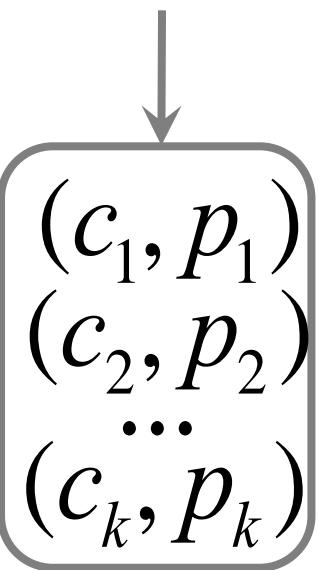
Further study of exact tests is motivated by the pessimism of existing sufficient tests

Runtime Evaluation



Pruning Constraint 3:

Sufficient Schedulability Condition [Baruah 2007]



$p_k - \bar{I}_k \geq 3 c_k$

Pruning Constraint 3: Sufficient Schedulability Condition



53 states (instead of 163)