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Abstract

This paper revisits the study of the Slotted ALOHA protocol with J = 2 terminals. Unlike previous approaches, this work employs multi-objective optimization tools. The work is focused on the characterization of the boundary (envelope) or Pareto optimal front curve of different types of trade-off region: the conventional throughput region, sum-throughput vs. fairness, and sum-throughput vs. transmit power. When possible, parametric and non-parametric expressions of these envelopes are here provided. Fairness is evaluated by means of the Gini-index, which is a metric used in economics to measure income inequality. Transmit power is directly linked to the global transmission rate. The approach presented in this paper generalizes previous works and provides more insights into the operation of random access protocols.

Trade-Off Performance Regions of Slotted ALOHA Protocol using Multi-Objective Optimization

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***Abstract*—This paper revisits the study of the textbook protocol Slotted ALOHA with $J = 2$ statistically different terminals. Unlike previous approaches, this paper employs multi-objective optimization tools. The work is focused on the characterization of the boundary (envelope) or Pareto optimal frontier of different types of trade-off region: the conventional throughput region, sum-throughput vs. fairness, and sum-throughput vs. transmit power consumption. Parametric and non-parametric expressions of these frontiers are here provided. Fairness is evaluated by means of the Gini-index, which is a metric used in economics to measure income inequality. Transmit power consumption is directly linked to the global transmission rate. The approach presented in this paper generalizes previous works and provides more insights into the operation of random access protocols in terms of trade-off performance design.**

***Index Terms*—S-ALOHA, random access, multi-objective optimization, Pareto optimal trade-off curve.**

I. Introduction

The ALOHA protocol lies at the core of the theory of *random access*. Since its proposal by Abramson in [1] and later in [2], ALOHA has been target of multiple reinterpretations. Recent approaches have reopened the analysis with advanced signal processing tools such as multi-packet reception (MPR) [3], cooperative diversity [4], and multi-hop ad-hoc features [5].

ALOHA has been mainly subject to *single-objective optimization* approaches (e.g. [7]). This paper addresses the *multi-objective optimization* of an *asymmetrical two-user Slotted ALOHA protocol*. Multi-user features can be inferred based on these results. However, full multi-user analysis is out of the scope of this paper[1]. The derivation of the boundary (envelope) of the throughput region is reformulated as the simultaneous optimization of two throughput functions. The envelope of the throughput region is identified as the Pareto optimal frontier. The remaining trade-off regions analysed are: *sum-throughput vs. fairness* and *sum-throughput vs. power consumption*. Power consumption is measured as the total transmission rate, which is an assumption commonly used in the study of random access (e.g. [9]). Fairness is evaluated by means of the Gini-index, which is used in economics to measure income inequality [10].

This paper is organized as follows. Section II describes the system model. Section III describes the performance metrics and the trade-off regions. Section IV addresses the multi-objective optimization. Section V presents sketches of the different regions, and finally Section VI presents conclusions.

[1]Some problems in this multi-user analysis have proved intractable with existing tools

II. System Model

Consider the slotted random access network depicted in Fig. 1 with one base station (BS) and $J = 2$ statistically different user terminals. Both users have their own buffer with packets assumed to be always available to be transmitted (*full queue or dominant system assumption*). At the beginning of every time-slot, each user $j \in \{1, 2\}$ will be assumed to attempt a transmission controlled by a Bernoulli random experiment with parameter p_j, which is also the *transmission probability* of terminal j. ALOHA operation is assumed: all packets involved in a collision will be considered to be lost.

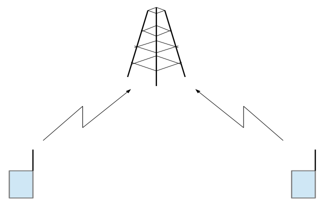

Fig. 1. Random access network with $J = 2$ users.

III. Trade-off Performance Regions

A. Throughput region

In ALOHA, the throughput per terminal (T_i) is equal to the probability of a single transmission to occur without collision. This can be mathematically written as follows:

$$T_1 = p_1\bar{p}_2 = p_1(1-p_2) \quad \text{and} \quad T_2 = p_2\bar{p}_1 = p_2(1-p_1). \quad (1)$$

Let us now define the concept of throughput region. Consider the vector $\mathbf{T} = [T_1 \quad T_2]$ of stacked throughput values of the two users, and the vector $\mathbf{p} = [p_1 \quad p_2]$ of stacked transmission probabilities. The throughput trade-off region, or simply the *throughput region*, can be defined as the union of all achievable throughput values $[T_1 \quad T_2]$ in (1) for all possible realizations of transmission policies ($0 \leq p_j \leq 1$) [6]:

$$\mathcal{C}_T = \{\tilde{\mathbf{T}} | \tilde{T}_j = T_j(\mathbf{p}), 0 \leq p_j \leq 1\}. \quad (2)$$

B. Sum-throughput vs. fairness region

Considering the individual throughput expressions in (1), the sum-throughput (T) is defined as follows:

$$T = T_1 + T_2 = p_1 + p_2 - 2p_1p_2. \quad (3)$$

Fairness will be evaluated by means of the Gini-index, which is commonly used to measure wage inequality [10]. The Gini-index is mathematically defined as follows [10]:

$$F_G = \frac{\sum_{j=1}^{J}\sum_{k=1}^{J}|T_j - T_k|}{2J^2\mu}, \tag{4}$$

where $\mu = \sum_{j=1}^{J} T_j/J$ is the mean value and $|\cdot|$ is the absolute value operator. A value of Gini-index equal to zero ($F_G = 0$) is the indication of *maximum fairness* where users are statistically identical. On the contrary, a value of Gini-index equal to one ($F_G = 1$) indicates the worst fairness scenario with one of the terminals using all network resources. For convenience in subsequent analysis, (4) can be rewritten as follows: $F_G = \frac{\sum_{j=1}^{J}\sum_{k=1}^{J} a_{j,k}(T_j - T_k)}{2JT}$, where $a_{j,k}$ is defined as $a_{j,k} = \begin{cases} 1, & T_j \geq T_k \\ -1, & T_j < T_k \end{cases}$. By considering the difference between throughput values: $T_1 - T_2 = p_1 - p_1p_2 - p_2 + p_1p_2 = p_1 - p_2$, the fairness indicator can be conveniently rewritten as: $F_G = \frac{|p_1 - p_2|}{T} = \frac{a_{1,2}(p_1 - p_2)}{T} = \frac{X}{T}$. Consider the vector $\mathbf{F} = [T \quad F_G]^T$ of stacked values of sum-throughput and fairness. The *sum-throughput vs. fairness trade-off region* ($\mathcal{C}_F$) can be defined as the union of all achievable values $[T \quad F_G]$ for all possible realizations of transmission policies ($0 \leq p_j \leq 1$):

$$\mathcal{C}_F = \{\tilde{\mathbf{F}} | \tilde{T} = T(\mathbf{p}), \tilde{F}_G = F_G(\mathbf{p}), 0 \leq p_j \leq 1\}. \tag{5}$$

C. Sum-throughput vs. transmit power region

Transmit power consumption will be considered here as proportional to the transmission rate, which is a common assumption in the literature (e.g., [9]). Therefore, we can define the average consumed power as:

$$P = \alpha(p_1 + p_2), \tag{6}$$

where α is a proportionality constant. Consider the vector $\mathbf{P} = [T \quad P]^T$ of stacked values of sum-throughput and power. The *sum-throughput vs. power trade-off region* ($\mathcal{C}_P$) can be defined as the union of all achievable values $[T \quad P]^T$ for all possible realizations of transmission policies ($0 \leq p_j \leq 1$):

$$\mathcal{C}_P = \{\tilde{\mathbf{P}} | \tilde{T} = T(\mathbf{p}), \tilde{P} = P(\mathbf{p}), 0 \leq p_j \leq 1\}, \tag{7}$$

IV. Optimization

A. Multi-objective optimization

To derive the envelope of the different trade-off regions, a multi-objective optimization method is here proposed, where the $M = 2$ objective functions F_m ($m = 1, \ldots, M$) can be simultaneously optimized:

$$\mathbf{p}_{opt} = \arg\max_{\mathbf{p}} \quad [F_1 \quad \ldots \quad F_M]. \tag{8}$$

Since this *vector optimization* usually lacks a unique solution [8], the concept of Pareto optimal frontier is commonly employed. A Pareto optimal solution can be loosely defined here as the point that is at least optimum for one or more of the elements of the vector objective function $[F_1, F_2 \quad, \ldots, F_m, \ldots \quad F_M]$ without degrading some of the other objective values (see [8] for a complete definition). The multi-objective optimization problem can be transformed into a single objective optimization problem using the method of scalarization [8]: $\mathbf{p}_{opt} = \arg\max_{\mathbf{p}} \sum_{m=1}^{M} \lambda_m F_m$, where λ_m is the relative weight given to the mth objective function. Differentiating the objective function we obtain a set of equations given by $\sum_{m=1}^{M} \lambda_m \frac{\partial F_m}{\partial p_k} = 0, \quad k = 1.., J$. The solution of this set of equations (assuming $J = M$) independent from the values of the weighting coefficients λ_m can be proved to be equivalent to setting a Jacobian determinant equal to zero [2] [11]:

$$|\mathbf{J}| = 0. \tag{9}$$

where the elements of Jacobian matrix $\mathbf{J}$ are given by $J_{m,k} = \partial F_m / \partial p_k$.

B. Throughput region ($\mathcal{C}_T$)

In the case of the throughput region, the two objective functions to be optimized are given by the throughput functions of each terminal ($F_1 = T_1$ and $F_2 = T_2$). In this context, (9) reduces to the well known result for the optimum transmission probabilities of ALOHA in asymmetrical settings:

$$p_1 + p_2 = 1. \tag{10}$$

By substituting this expression in the throughput expressions in (1) we obtain $T_1 = p_1^2$ and $T_2 = p_2^2$, which substituted back in the previous expression in (10) yields $\sqrt{T_1} + \sqrt{T_2} = 1$. This is the non-parametric expression[2]of the envelope of the throughput region. For convenience, this expression can be algebraically manipulated as follows $(T_1 - T_2)^2 - 2(T_1 + T_2) + 1 = 0$, which is the equation of a parabola rotated 45 degrees with respect to coordinate system (T_2 vs. T_1) with positive concavity (i.e., $\frac{\partial y^2}{\partial x^2} < 0$, where $x = T_2 - T_1$ and $y = T_1 + T_2$).

C. Sum-throughput vs. fairness region ($\mathcal{C}_F$)

Consider that the two objective functions to be optimized are $F_1 = F_G$ and $F_2 = T$. For convenience, let us now express the first-order derivative of F_G with respect to p_j as follows: $\frac{\partial F_G}{\partial p_j} = \frac{\partial T}{\partial p_j}\frac{1}{T^2}\left(\frac{T\frac{\partial X}{\partial p_j}}{\frac{\partial T}{\partial p_j}} - X\right)$. Using the properties of determinants, the expression in (9) reduces to:

$$\frac{\partial X}{\partial p_1}\frac{\partial T}{\partial p_2} = \frac{\partial X}{\partial p_2}\frac{\partial T}{\partial p_1}, \tag{11}$$

where: $\frac{\partial X}{\partial p_1} = a_{1,2}$, $\frac{\partial X}{\partial p_2} = -a_{1,2}$, $\frac{\partial T}{\partial p_1} = 1 - 2p_2$, and $\frac{\partial T}{\partial p_2} = 1 - 2p_1$. By substituting these expressions in (11) we obtain:

$$p_1 + p_2 = 1. \tag{12}$$

Note that the optimum solution for the case of the envelope of the $\mathcal{C}_F$ region is identical to the functional solution for the case of the throughput region in (10). Let us now substitute this expression back in the expressions for fairness and sum-throughput, which after some convenient modifications are given, respectively, by $F_G T = a_{1,2}(2p_1 - 1)$ and $T = 2p_1^2 - 2p_1 + 1$. These expressions can be rewritten in non-parametric form as follows $2T = \left(\frac{F_G T}{a_{1,2}}\right)^2 + 1$.

[2]In this paper, parametric expressions are explicit functions of the transmission probabilities p_j. Non-parametric expressions are functional relationships exclusively in terms of the metrics of the trade-off region under investigation.

D. Sum-throughput vs. transmit power region ($\mathcal{C}_P$)

Consider that the two objective functions are now given by $F_1 = P$ and $F_2 = T$, then (9) can be proved to reduce to:

$$p_1 = p_2 \tag{13}$$

By substituting this result back in the expressions for sum-throughput and average power we obtain, respectively $P = 2\alpha p_1$ and $T = 2p_1 - 2p_1^2$, which can be rewritten in non-parametric form as follows: $\left(\frac{P}{\alpha} - 1\right)^2 + 2T = 1$. This is the equation of a parabola aligned with the axis of the $\mathcal{C}_P$ region (T vs. P) with negative concavity (i.e., $\frac{\partial T^2}{\partial P^2} < 0$).

V. Results and discussion

A. Throughput region

Fig. 2 presents the sketches of the throughput region $\mathcal{C}_T$ in (2) whose envelope is given in non-parametric form by boundary conditions ($\{T_1, T_2\} \in \{0, 1\}$) and by the expressions derived in subsection IV-B. The parametric form is given by the throughput expressions in (1) and the optimum transmission policy in (10). It can be observed that the $\mathcal{C}_T$ region has a non-convex shape, where any reduction of the performance of one of the users is considerably higher than any potential performance gain achieved by the other user. The main envelope of the throughput region has a parabolic shape as shown in subsection IV-B (tagged in Fig. 2 as $p_1+p_2 = 1$). Each user throughput function is maximized when its transmission probability equals one and when the transmission probability of the other user equals zero. When both users achieve exactly the same throughput ($T_1 = T_2 = 0.25$) along the the envelope of the throughput region, they do so with a transmission probability equal to $p_1 = p_2 = 0.5$. The line that connects this point with the origin of the coordinate system (T_2 vs. T_1) represents the rotation axis (at 45 degrees with respect to the coordinate system) of the parabola with positive concavity that defines the boundary of the $\mathcal{C}_T$ region (i.e., $p_1+p_2 = 1$). This straight line is tagged in Fig. 2 as $p_1 = p_2$.

B. Sum-throughput vs. fairness region

Fig. 3 presents the sketches of the $\mathcal{C}_F$ region in (5) whose envelope is given in non-parametric form by boundary conditions: $(T, F_G) \in \{0, 1\}$, and by the expressions derived in subsection IV-C. The parametric form is given by the expressions for fairness in (4), sum-throughput in (3), and the expression for the optimum transmission policy in (12). The trade-off region is shown to be upper bounded by exactly the same functional relationship that defines the main bound of the throughput region ($p_1+p_2 = 1$). The point with maximum sum-throughput and worst Gini index at the top right corner of the figure is given by two operational points, which correspond to the cases where one of the users transmits with probability one while the other is idle (zero transmission probability). These points are $(T_1, T_2) = (1, 0)$ and $(T_1, T_2) = (0, 1)$, which are also given by $(p_1, p_2) = (1, 0)$ and $(p_1, p_2) = (0, 1)$, respectively. We recall here that a value of Gini-index $F_G = 1$ indicates the worst fairness case where one of the terminals uses all network resources. On the other hand, a value of Gini index $F_G = 0$ indicates the maximum fairness between the users who achieve identical statistical performance. The point with the best fairness ($F_G = 0$) and maximum sum-throughput ($T = 1$) at the top left corner of the figure is given by the case where both users experience the same throughput ($T_1 = T_2 = 0.25$) with the same transmission probability $p_1 = p_2 = 0.5$. The left side boundary of the region, given by the best fairness indicator ($F_G = 0$), corresponds to the curve with equal throughput ($T_1 = T_2$), which is also the curve with equal transmission probabilities for both users ($p_1 = p_2$). Note that the projection of this equal throughput solution in the $\mathcal{C}_T$ region is the rotation axis of the parabolic function that defines the main bound of the throughput region, as shown in the previous subsection. An interesting feature is observed at the bottom boundary of the trade-off region, which can be proved to be undetermined for all the values except for the origin[3]. The origin corresponds to the point with maximum fairness and sum-throughput equal to zero ($F_G = 0$ and $T = 0$), which is given by two cases: when both users have no transmissions (zero transmission probability: $p_1 = p_2 = 0$), or when both users transmit with probability one ($p_1 = p_2 = 1$).

C. Sum-throughput vs. power region

Fig. 4 presents the sketches of the $\mathcal{C}_P$ trade-off region in (7) whose envelope is given in non-parametric form by boundary conditions: $(p_1, p_2) \in \{0, 1\}$, and by the expressions derived in subsection IV-D. The parametric form is given by the sum-throughput expression in (3) and the expression for the optimum transmission policy in (13) ($p_1 = p_2$). All the results assume a unitary value for the proportionality constant $\alpha = 1$ in (6). The region is defined by three points. The first one is the origin, which corresponds to the case where none of the users transmits information ($p_1 = p_2 = 0$), and which leads to zero sum-throughput and also zero-power consumption ($T = P = 0$). The second point is the solution with maximum sum-throughput ($T = 1$), which corresponds to the two cases where one of the users transmit with probability one and the other user transmits with zero probability ($p_1 = 1$ and $p_2 = 0$ or $p_1 = 0$ and $p_2 = 1$). The third point is the solution with maximum power consumption ($P = 2$) and with zero sum-throughput ($T = 0$), which is given by the case where both users transmit with probability one ($p_1 = p_2 = 1$). The bottom boundary of the region is defined by the curve in (13) with equal transmission probabilities ($p_1 = p_2$) which has a parabolic non-parametric shape with negative concavity. The projection of this bound coincides with the maximum fairness boundary of the $\mathcal{C}_F$ region, and also with the rotational axis of the parabolic function that defines the main bound of the $\mathcal{C}_T$ region. The line that defines the top left boundary of the region is defined by two possible cases where one of the users remains silent (zero transmission probability), while the other user adopts any transmission probability (i.e., $p_1 = 0$ or $p_2 = 0$). The line that defines the right top boundary is the mirror of the line previously described, and it is given by two possible cases where one of the user keeps transmitting with

[3]Note that there is no point in the solution space that complies with $T = 0$ and $F_G > 0$

probability equal to one ($p_1 = 1$ or $p_2 = 1$) and the other terminal adopts any value of transmission probability. Note that the projection of the optimum transmission policy of the $\mathcal{C}_T$ and $mathcalC_F$ regions is a constant power line cutting into two halves the $\mathcal{C}_P$ region ($p_1 + p_2 = 1$).

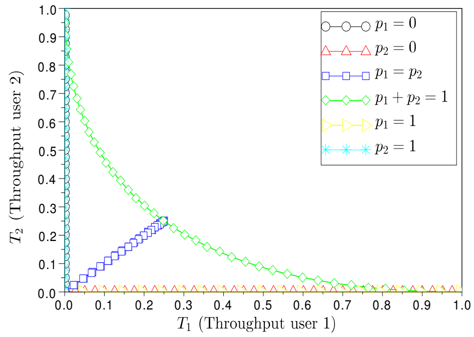

Fig. 2. Throughput region ($\mathcal{C}_T$).

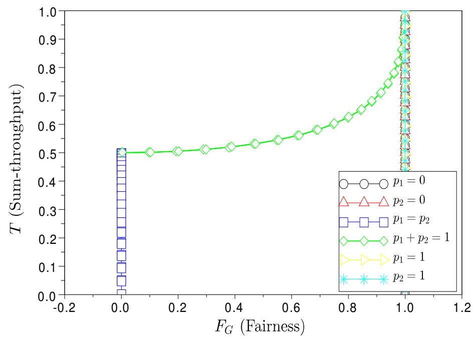

Fig. 3. Sum-throughput vs. fairness trade-off region ($\mathcal{C}_F$).

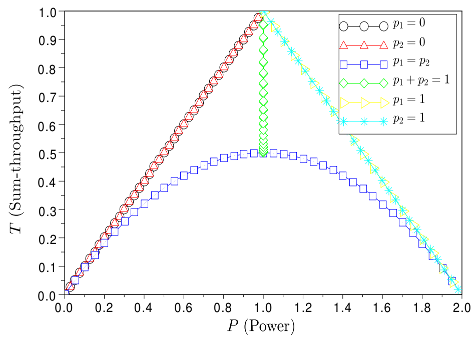

Fig. 4. Sum-throughput vs. power consumption trade-off region ($\mathcal{C}_P$).

VI. Conclusions

This paper has presented a trade-off analysis of different types of metrics of a two-user S-ALOHA protocol using multi-objective optimization. The trade-off regions investigated were: throughput region, sum-throughput vs. fairness, and sum-throughput vs. power consumption. The boundaries of these regions were derived in parametric and non-parametric form. The main boundary or envelope of the throughput region was found to be described by a parabola rotated 45 degrees with respect to the coordinate system of the throughput region. The projection of this curve was found to also describe the top boundary of the fairness region. In addition, this same curve was found to be a constant power curve that cuts in half the sum-throughput vs. power trade-off region. The straight line at 45 degrees that represents the rotation axis of the parabola that defines the top boundary of the throughput region was also found to be the equal transmission probability curve (and therefore the maximum fairness boundary of the system), as well as the minimum sum-throughput bound of the sum-throughput vs. power trade-off region. This line was also found to be described by a parabola aligned with the y-axis of the coordinate system of the sum-throughput vs. power region with negative concavity. The bottom boundary of the sum-throughput vs. fairness region was found to be asymptotically undetermined, and its projection to be a single point in the origin of the throughput region of the system. These results provide more details on the intricate relationships between different metrics and parameters of random access protocols, providing a framework for future analysis and interpretation of more complex networks.

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